Taylor's power law for a random network

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Abstract

Taylor's power law states that the variance function is quadratic. It is observed for population densities of species in ecology. Taylor's power law is called after the British ecologist L. R. Taylor (see [4]). For random networks another power law, that is the power law degree distribution is widely studied (see Barabási and Albert [1]). In this paper a precise mathematical proof is presented that the original Taylor's power law is asymptotically true for the N-stars network evolution model.

We call a graph N-star graph if it has N vertices, one of them is called central vertex, the remaining $N-1$ vertices are called peripheral vertices, and they are connected to the central one. The N-star network evolution model is the following (see [2]). At each step either a new N-star is constructed or an old one is activated again. The central weight of a vertex is w_1, if the vertex was w_1 times central vertex during the activations. The peripheral weight of a vertex is w_2, if the vertex was w_2 times peripheral vertex during activations. In this paper we calculate the mean and the variance of w_2 when w_1 is fixed, and we shall see that the variance function is asymptotically quadratic (see [3]).

Keywords: Taylor's power law, random graph, preferential attachment, scale free, gamma function

MSC: 05C80, 62E10

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