Elementary Computational Exploration of the Degree Sequence of the Malyshev Polynomials

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Abstract

Zolotarev's First Approximation Problem (ZFP), which is one of Kaltofen's favorite open problems in symbolic computation, asks to select the one among all monic polynomials of fixed degree $n >= 2$ and fixed 2nd leading coefficient $a_{n_1} = -ns$ ($s \in R^+$) which deviates the least from zero on the interval $I = [-1, 1]$. I turns out that this extremal polynomial also deviates the least among the monic polynomials of fixed degree n on the set which consists of two disjoint intervals $S = I \cup (\alpha(s), beta(s))$, $beta > \alpha > 1$, and can be characterized uniquely by roots of bivariate integer polynomials $F = F(s, \alpha)$, $G(s, \beta)$ (and their roots). We coined these polynomials Malyshev polynomials, since Malyshev was the first who systematically enumerated these polynomials in 2002 [Malyshev2002] up to degree 5. In this paper we investigate the degree sequence of F and G via computational tools up to degree 16 and come up to a general form. The obtained degree sequence coincides with Sequence A055932 in the OEIS database. We explain the obtained results via the connection of the Malyshev polynomials to Schiefermayr's asymmetric homogeneous 4-variate polynomials for T-polynomials and to the generalized Zolotarev polynomials. For the computations we used the computer algebra systems Maple and Mathematica.

Keywords: Abel-Pell differential equation, degree sequence, extremal polynomial, Malyshev polynomial, symbolic computation, T-polynomial, Zolotarev's First Problem, Zolotarev polynomial