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The BWConverter Toolchain: An Incomplete Way to Convert SAT Problems into Directed Graphs

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Abstract

In our previous works we showed, how to convert a directed graph into a SAT problem [3, 12]. We have presented two SAT solvers [4, 10, 11] and introduced several models [13]. We showed that if the directed graph is strongly connected, then the converted SAT problem is Black-and-White. A SAT problem is Black-and-White if it has exactly two solutions, the black assignment, where each variable is false, and the white one, where each variable is true. In this poster, we study the other way: How to convert a SAT problem into a directed graph. This direction seems to be very difficult, because if we would have a complete converter from SAT to directed graphs, then we could use our previous result to convert it to 2-SAT, i.e., we would have a SAT to 2-SAT converter. What we present is just an incomplete converter, so we do not attack the **P** = **NP** question. The **BWConverter toolchain** contains two tools: the **BWConverter** and the **NPP** tool. As a first step, one has to use the BWConverter tool, which creates a Black-and-White SAT problem from any UNSAT problems. The BWConverter makes sure that its output has the same set of solutions as the input, except the black and the white assignment. To do so, we have implemented several algorithms. The most interesting one introduces two new variables, X and Y; then it adds $(\neg Y)$ to each clause which contains only positive literals; then it adds (X) to each clause which contains only negative ones; finally, it adds $(\neg X \vee Y)$ as a new clause to the output. The resulting SAT problem is Black-and-White if the input is UNSAT. As a second step, we generate a directed graph using our NPP tool. This tool generates edges from clauses which contain either exactly one positive literal, or exactly one negative one using the observations of our previous models. From example from the clause $(\neg A \vee B \vee C)$ it generates the edges: $A \to B$ with weight 50%, and $A \to C$ with weight 50%. From the clause $(\neg A \vee \neg B \vee C)$ it generates the edges (the weights are in ()): $A \to C$ (50%), $B \to C$ (50%), $A \to B$ (50%), and $B \to A$ (50%). This conversation is too eager, because it creates a strongly connected graph even if the input was not a Black-and-White SAT problem. As a solution, we attach to each edge a weight number which indicates, how likely is that the edge is important. Then we iteratively remove the edges with the smallest and / or with the highest weight until we would a non strongly connected directed graph in the next iteration. We call this directed graph to be the directed graph representation of the input SAT problem. We studied where this state transition point is in case of unsatisfiable random 3-SAT problems.

Introduction

The propositional satisfiability problem, for short the SAT problem, is the the problem to find an assignment to the the Boolean variables of the input formula which makes it true. The formula is given in conjunctive normal form which is represented as a clause set. If each clause consists of at most k literals, then it is a k-SAT problem. We know that the 2-SAT problem is solvable in linear time [1]. On the other hand 3-SAT is **NP-complete** [7]. So, there is a big the gap between the easiness of 2-SAT and hardness of 3-SAT.

Horn SAT is the restriction to instances where each clause has at most one positive literal. Horn SAT is solvable in linear time [9], as are a number of generalizations such as renamable Horn SAT [2], extended Horn SAT [6] and q-Horn SAT [5]. The hierarchy of *tractable* satisfiability problems [8], which is based on Horn SAT and 2-SAT, is solvable in polynomial time. An instance on the k-th level of the hierarchy is solvable in $O(n^{k+1})$ time. *r,r-SAT*, where r,s-SAT is the class of problems in which every clause has exactly r literals and every variable has at most s occurrences. All r,r-SAT problems are satisfiable in polynomial time [15]. A formula is SLUR (Single Lookahead Unit Resolution) *solvable* if, for all possible sequences of selected variables, algorithm SLUR does not give up. Algorithm SLUR is a non-deterministic algorithm based on unit propagation. It eventually gives up the search if it starts with, or creates, an unsatisfiable formula with no unit clauses. The class of SLUR solvable formulae was developed as a generalization including Horn SAT, renamable Horn SAT, extended Horn SAT, and the class of CC-balanced formulae [14]. In this poster we restrict the general SAT problem such that each clause should contain at least one positive and one negative literal. We study how can we generate a directed graph from this kind of problem. Our goal is to give tools which help this work.

The BWConverter tool

The BWConverter tool creates a Black-and-White SAT problem from an UNSAT problem. It has 3 main switches:

```
BWConverter -nooverlap num  |  BWConverter -n num
BWConverter -overlap num  |  BWConverter -o num
BWConverter -pigeonholeeater  |  BWConverter -p
```

After the first two ones we give a number, which shows how smart do we use these strategies. They remove the black and the white clauses from each input clause. We introduce them by examples. Let us assume that the input has 6 variables: from a to f, these clauses was already processed: $\{a, d, e\}$, $\{b, c, d\}$, and the current clause is $\{a, b, c\}$. Then:

-n 0 generates: $\{a, b, c, \neg d\}$, $\{a, b, c, d, \neg e\}$, $\{a, b, c, d, e, \neg f\}$.
-n 1 generates: $\{a, b, c, \neg d\}$, $\{a, b, c, d, \neg e\}$.
-n 2 generates: $\{a, b, c, \neg d\}$.
-o 0 generates: $\{a, b, c, \neg d\}$, $\{a, b, c, \neg e\}$, $\{a, b, c, \neg f\}$.
-o 1 generates: $\{a, b, c, \neg d\}$, $\{a, b, c, \neg e\}$.
-o 2 generates: $\{a, b, c, \neg d\}$.

-p introduces two new variables as described in the abstract.

The NPP Tool

The NPP tool work as follows: it reads a CNF file in DIMACS format. For each clause C it adds the edges (n, p, w) to the graph, where n is a negative literal, p is a positive literal, and w is $1/pos$, where pos is the number of positive literals in C. It also adds the edges (n_1, n_2, nw) to the graph, where n_1 and n_2 are different negative literals, and $nw = 1/neg$, where neg is the number of positive literals in C.

For example if $C = \{\neg a, \neg b, \neg c, d, e\}$, then the following edges is added to the graph: $(a, d, 0.5)$, $(b, d, 0.5)$, $(c, d, 0.5)$, $(a, e, 0.5)$, $(b, e, 0.5)$, $(c, e, 0.5)$, $(a, b, 0.33)$, $(a, c, 0.33)$, $(b, a, 0.33)$, $(b, c, 0.33)$, $(c, a, 0.33)$, $(c, b, 0.33)$.

The tool has the following switches:

```
NPP -autominlimit  |  NPP -a
NPP -automaxlimit  |  NPP -A
NPP -setminlimit num  |  NPP -s num
NPP -setmaxlimit num  |  NPP -S num
```

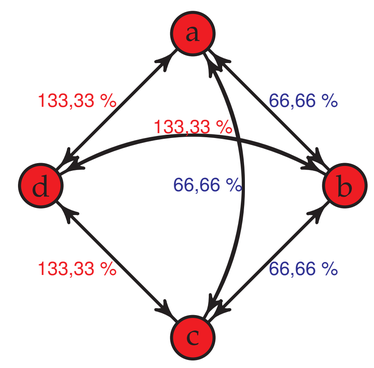
-a finds the biggest limit, such that by deleting edges with weight $\leq$ limit the directed graph is still strongly connected. ***-A*** finds the lowest limit, such that by deleting edges with weight $\geq$ limit the directed graph is still strongly connected. ***-s*** sets the lower limit to num, i.e., it deletes all edges where the weight $\leq$ num. ***-S*** sets the upper limit to num, i.e., it deletes all edges where the weight $\geq$ num.

The most typical usage is ***NPP -a -A*** which finds automatically both limits.

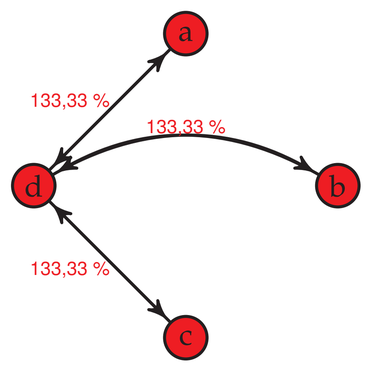
An Example

The initial **unsatisfiable** 3-SAT formula let be as follows:
$[(\neg a \vee b \vee d) \wedge (a \vee \neg c \vee d) \wedge (a \vee \neg b \vee \neg d) \wedge (b \vee \neg c \vee \neg d) \wedge (\neg a \vee c \vee \neg d) \wedge (a \vee b \vee c) \wedge (\neg a \vee \neg b \vee \neg c)]$

After running the BWConverter tool, the modified formula is as follows:
$[(\neg a \vee b \vee d) \wedge (a \vee \neg c \vee d) \wedge (a \vee \neg b \vee \neg d) \wedge (b \vee \neg c \vee \neg d) \wedge (\neg a \vee c \vee \neg d) \wedge (a \vee b \vee c \vee \neg d) \wedge (\neg a \vee \neg b \vee \neg c \vee d)]$ Note, that this is not **unsatisfiable** any more, it is Black-and-White.



The generated graph from the formula



Result of using the tool with -a -A (NPP -a -A)

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Usage and Conclusion

The toolchain can be found here: http://fmv.ektf.hu/tools.html It consists of two tools: BWConverter, and NPP. Its typical usage is the following:

- `java BWConverter -o 2 unsat.cnf bw.cnf`
- `java NPP -a -A bw.cnf`

This work was an inner project, because we used to need a tool to test our theories. For us the toolchain was very usefull, so we decided to publish it. As future work we should prove the lemmas what we found by the help of this toolchain.