

The Joy of Queueing Theory

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Abstract

The main goal of the present article is to introduce some software packages which help with understanding Queueing Theory and to show how our recently developed application called Queueing Systems Assistance (QSA) works. This package is combined with a lecture note with the aim of calculating and demonstrating the main operational characteristics of queueing systems. Furthermore, it enables us to find the minimum of a quite general average total cost per unit of time with a linear cost function. Several sample examples are collected to show the advantage of the graphical module built into the application

1. Introduction

I have been teaching Queueing Theory (QT) and its applications since 1978 thus I have the experience to realize that teaching materials and software need innovation and new methods to attract the attention of the students. The applications area has changed a lot recently, and I am sure that more and more students and practitioners need to use QT. The increased computational capacity of modern devices has greatly contributed to the visualization of results and thus helps us in a better understanding of the theory. In my lecture note, [9] I treated several fundamental queueing systems that are useful in investigating a wide variety of stochastic systems occurring in different service processes. I think that there is a need for such material in view of the increased use of queueing models in modern technology. The application of queueing theory in the performance analysis of computer and communication systems has stimulated much practically oriented research on computational aspects of queueing models. To use the formulas derived in the note, a software package called QSA (Queueing Systems Assistance)

was developed in 2021 and it is embedded into my lecture note [9] with the purpose to calculate and visualize the principal operational characteristics. In addition, it helps to minimize a quite general mean total cost per unit of time with a linear objective function. The greatest advantage of this software is that these scripts can run on all modern devices including smartphones, too, thus the software is very comfortable for students and improves the effectiveness of a teacher. To deal with problems that occurred in different fields of application the initial step is to identify the appropriate queueing system and then to determine the performance measures. Of course, the mathematical level of modeling greatly depends on the distribution functions of the involved random variables, such as the time between arrivals, and service times. My recommendation is, to begin with, a simple system and then if the results do not fit in with the problem carry on with a more complicated one. Different software tools offer solutions to interested readers on various levels.

We propose the next link to visit where the tools are collected

<http://web2.uwindsor.ca/math/hlynka/qsoft.html>

We have found some fundamental books on QT in which software support was provided, for example, Mathematica in [1, 4], MatLab in [2, 5–7].

The usage of spreadsheets is good advice for calculations of the formulas. Our recommendation is an Excel-based tool called QTSPPlus to determine the principal operational characteristics of the related queueing systems. It is connected to the book of Gross, Shortle, Thompson and Harris [3] and can be obtained from the following link

ftp://ftp.wiley.com/public/sci_tech_med/queueing_theory/

2. Queueing Systems Assistance (QSA)

For problem-solving-oriented teaching lectures we have also developed our own software tool called QSA (Queueing Systems Assistance) see, Szilagyi, et. al. [8] to calculate and visualize the operational characteristics of the systems together with optimal decisions not only for elementary but more advanced queueing systems as well. It is available at

<https://qsa.inf.unideb.hu>

The main advantages of QSA over QTSPPlus are the following

- It runs on desktops, laptops, and mobile devices @It determines not only the average but the variance of the random variables in question
- It derive the distribution function of the waiting/response times (if possible)
- It demonstrates all the main operational characteristics
- It supports decision-making graphically

QSA is a user interface, a web-based application written in TypeScript. Any browser (Firefox, Chrome, Edge, etc.) on every platform (Windows, Linux, Android, iOS) is supported, which means one can apply mobile and desktop devices for performing any calculations which are executed on the server. There are no hardware limitations, and the source code is available on GitHub, under the MIT license, so anyone interested can check out the code or help to develop the application. QSA is embedded into the lecture note of Sztrik [9].

One of the special features of the software is that the operational characteristics of M/G/1/K/K systems with deterministic, Erlang, Hypo-exponential, Hyper-exponential, and gamma distributed service times are determined. The distribution function of the waiting/response times of the M/M/c/K, M/M/c/m/K systems and the operational characteristics of M/M/c/K, M/M/c/m/K with balking and reneging are calculated as well. It was our goal to derive, where it is possible, the distribution function of the waiting/response time to solve decision problems. Furthermore, not only the average but the variances of the measures are calculated. What is also unique is the calculation of the mean total cost per unit of time in a steady state.

Since its release in February 2021, it has been used more than 48 000 times all over the world.

References

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