

Analysis of retrial queueing system with two-way communication and impatient customers using simulation

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Abstract

Two-way communication systems are often a popular topic because they can be modeled using retrial queueing systems in many areas of life. One noteworthy example is the operation of call centers where agents perform other activities, such as selling products or advertising, while customers are on hold. Critical to the success of this system is utilization—how best to optimize service units (agents) and make sure that each one provides optimal customer experience, see for example [2],[4]. Past research works have also investigated infinite queueing systems with two-way communication; here are a few examples: [1],[5],[6]. Two-way communication, which is essential for server maintenance and operation, relies on calls being placed both inside the system and outside of it when the server is idle. These calls are referred to as outgoing calls. In our model, outgoing calls are allowed to be made to either the source or orbit.

Some queueing models make the assumption that a consumer must wait in line indefinitely before being serviced. When a customer enters and discovers the service area is occupied, some additional models—known as loss models—have the customer leave and lose him or her forever. However, there are countless situations in real life where customers choose to attempt to be served after an arbitrary amount of time has passed rather than waiting. In this scenario, the client waits in a virtual waiting area called an orbit before making another attempt to contact the server. Retrial queues can be used to represent models that have an orbit. Queueing systems with retrial queues are frequent and effective modeling tools for issues that arise in important telecommunications systems, including as call cen-

ters, wireless mesh networks based on CSMA, and telephone switching systems. Their significance is demonstrated in the works listed below, such as [3].

The originality of this work lies in the sensitivity analysis that was conducted to examine how different retrial time distributions affect the key performance metrics. Our stochastic simulation program, which is based on SimPack produces the results. To support discrete event simulation, continuous simulation, and combined (multi-model) simulation, this is a collection of C/C++ libraries and executable programs. Any sort of queueing system and simulation model can be freely modeled, and any performance metric can be calculated using any random number generator for the specified random variable. The comparison of the operating modes and various distributions will be shown through graphical representations.

The system model

This section introduces the finite-source retrial queueing paradigm with a single server under consideration. The source contains a total of N requests, each of which can produce a primary incoming call to the server. The inter-request times are determined by exponentially distributed random variables with the parameter λ_1 . When the server is idle, an incoming customer's service starts immediately and follows an exponential distribution with parameter μ_1 . After receiving satisfactory service, clients return to the original provider. Customers who arrive and find the service unit busy will not be lost; instead, they are transported to orbit. These are the secondary arriving jobs from the orbit that might make another attempt to contact the service unit following an arbitrary waiting period. Gamma, hyper-exponential, Pareto, and lognormal distributions are all used to describe this period's distribution, albeit they all have the same mean value. But the idle server may also request calls from the orbit and the source. We distinguish between two categories of outgoing calls:

- After an exponentially varying amount of time, the service unit may request a primary outgoing call from the source to be served λ_2 ,
- After an exponentially distributed period, the service unit may make a call (secondary outgoing call) from orbit ν_2 .

The outgoing customers' service time is distributed exponentially with the parameter μ_2 . When an incoming call is received from the orbit, there are two distinct scenarios:

- Case 1: After the outgoing service is complete, the call is returned back to the orbit to have its incoming call served because it has an unmet incoming request,
- Case 2: Here, the call also has an incoming request that hasn't been fulfilled, but as soon as the outgoing service is complete, the service unit fulfills the incoming request. A two-phase service will result from this, with the outgoing

call being processed first and then the incoming one. When both service phases have been completed, the call goes back to the source.

Every primary customer has an impatience trait, and in our investigated model a primary customer eventually departs the system after waiting in the orbit for some time without obtaining the proper service, which is also an exponentially distributed random variable with rate τ . The arrivals of primary incoming calls, retrial intervals for secondary incoming calls, service times for incoming and outgoing calls, and the amount of time needed to make outgoing calls are all considered to be independent of one another.

Utilizing this model, our goal is to perform a sensitivity analysis on the main performance measures using several distributions of retrial times. A number of system properties are compared between various operating modes as well. We developed a simulation program to get the results, which will be shown in a series of graphs.

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