Time Series in Practice; Going beyond linearity and stationarity

Gy. Terdik[a], **Á. Vinkó**[c], **N. Liegner**[c], **P. Bocz**[c], **Sz. Fischer**[b*]

[a]Department of Information Technology, Faculty of Informatics, University of Debrecen
terdik.gyorgy@inf.unideb.hu

[b]Department of Transport Infrastructure and Water Resources Engineering, Faculty of Architecture, Civil and Transport Engineering, Szechenyi Istvan University

[c]Department of Highway and Railway Engineering, Faculty of Civil Engineering, Budapest University of Technology and Economics

Abstract

"One can FT anything—often meaningfully."
—John W. Tukey [1]

In this paper we show the usefulness of EMD for detecting the trend and major frequencies for a nonlinear and non-stationary signal in case of a data set of irregularities in track geometry. The observations in everyday practice are neither linear nor stationary. In case the nonlinear model is a priori known then it is used for the further study otherwise some preliminary transformations can be applied for approximation to either linearity or stationarity or both. A frequently used transformation is the seasonal-trend (polynomial) decomposition which makes possible of usage ARIMA models, see Figure 1a. One can also use change point detection for the data and get rid of different means, variances etc. between change points Figure 1b.

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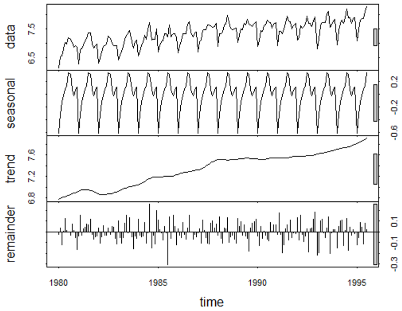
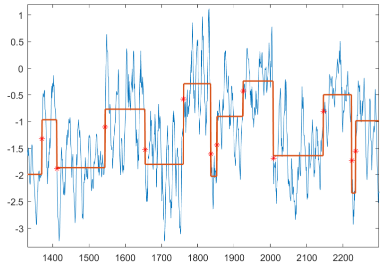


(a) Red wine sales.



(b) Change points and trend.

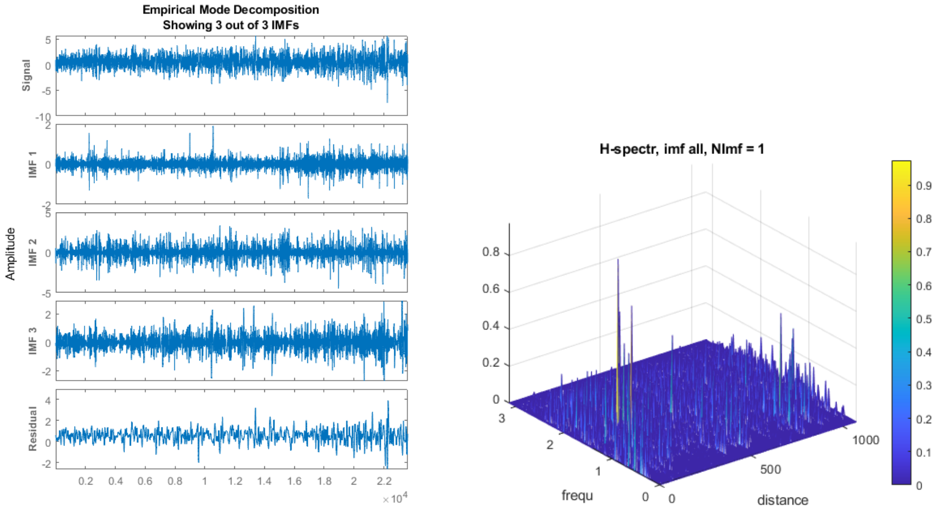
Figure 1

Measurement, Hilbert transform, EMD and Trend

We consider a series of measurements which concerns to the problem of characterizing of irregularities in track geometry. The data has been produced by a comprehensive track recording vehicle. The irregularity of the track gauge and the alignments of the right and left rails has been measured with the sampling interval of 0.25m along the track, see [2] for details. In this work we consider the measurement according to the right alignment with sample size 23,601. The Hungarian National Standard defines classes D1-3 with wavelength range 3m-200m, here the Class D1 with wavelength range 3m-25m is investigated for information on the wavelength and local distribution of the geometry faults. We address two basic issues: *One* is separate the trend and the random part of the signal. The *other one* is to find the characteristic wave lengths for different track sections.

The basic method is the Hilbert Transform (HT), [3]. The HT maps the temporal-space data to time-frequency space, where both the amplitudes and frequencies depend on time as well. The marginal Hilbert spectrum can be computed from the Hilbert spectrum, it collects the total amplitude contribution to each frequency value.

The Empirical Mode Decomposition (EMD) is a data-driven auto-adaptive method, which decomposes signals into components referred to as Intrinsic Mode Functions (IMF) and a residual. In our calculations we apply the most popular version which is implemented in MATLAB, see [4]. The IMFs fluctuate around zero. The only non-zero mean component is the remaining residual, see Figure 2a. The Hilbert spectrum $H_k(\omega, t)$ of an IMF $u_k(t) = A_k(t)\cos(\phi_k(t))$, where $\omega(t) = d\phi(t)/dt$, is defined by $H_k(\omega, t) = 0$ except $H_k(\omega(t), t) = A_k(t)$, see Figure 2b. We apply the methodology of [5] for the separation the trend from the signal, see Figure 3.



(a) Decomposition of the signal. **(b)** Hilbert spectrum of the first IMF.

Figure 2. IMF

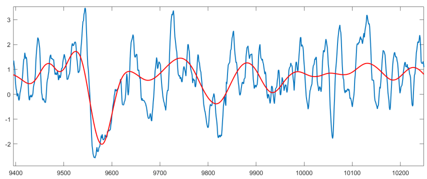


Figure 3. The measured signal and the trend.

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