ICAI 2023
12th International Conference on Applied Informatics
Eger, Hungary, 2023

Analysis of multi-source queueing systems with multiple servers using analytical and simulation programs

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Abstract

Queuing theory is one area of applied probability theory. Queuing can occur in many areas of life when a request cannot be serviced immediately by the service unit. Decisions in connection to the amount of capacity are very complicated and often impossible to make because it is unpredictable when customers will arrive or how much time will be needed to provide its appropriate service. With the help of queueing theory, various system characteristics are predictably lessening the hardship of such decisions providing important information about the performance measures.

Basic system model

In this section, we present the basic considered system model which is a multi-source queuing system with several server units (cabins) [6], [7]. This system models a spa where there are three different ways to buy a ticket:

- The first is online, where the potential customer buys a ticket through a website according to an exponential distribution with parameter λ_1.
- The second option is the obvious method, where the customer visits the spa area and buys a ticket at the cash desk before entering. If all the cabins are occupied then the customers will form a queue waiting for one of the cabins to be released. In our programs, the maximum number of queue lengths was limited to a specific number, since the probability of someone

joining the queue is low for long queue lengths. The arrival of the incoming customers to the cashier are also exponentially distributed random variables with parameter λ_2.

- The third way to get spa tickets is by booking a hotel room. In this case, the hotel can offer its guests discounted tickets as a service, which they can use during their stay. These are obviously bought in advance by the hotel so the third source will be finite. From here, those customers will arrive according to an exponential distribution with the parameter λ_3 and the maximum number of customers in the source is denoted by N. The number of serving units is M and their serving intensity is exponentially distributed with parameter μ. If all the servers are busy (all the cabins), the customer from the cashier is queued, a customer trying to request a ticket online cannot proceed to buy it, while the customer from the hotel returns to the finite source. Upon leaving the beach (when the request is served), the customer is returned to the source.

All random variables involved in the construction of the model are assumed to be completely independent of each other. This is our basic model which is upgraded by the orbit. These customers stay in the system and spend their time in a virtual waiting room called an orbit (around the spa area). Customers in the orbit try to get a ticket again after an exponentially distributed time with the parameter of ν. This can be modeled by a finite or infinite source retrial queueing model [4], [1], [3]. Several other working models can be considered. One possible scenario with an orbit can be seen in Figure 1.

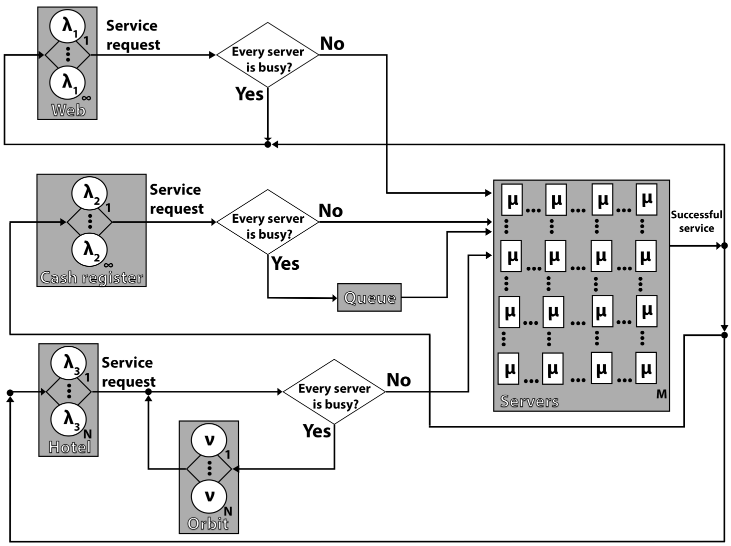

Figure 1. The system model

The main goal of our work is to carry out a sensitivity analysis to examine how different service time distributions affect the key performance metrics. For this, we use an analytical software, the MOSEL-2 tool [2], and a simulation tool, the SimPack [5]. MOSEL has strict limitations, e.g., it can handle only exponentially distributed random times. We solve a simplified model by MOSEL and SimPack, as well, and the identity of the two solutions validate the model. After that the SimPack can be used for more complex models with different distributions of random times. Thus, a large portion of the results are based on our stochastic simulation program, which is based on SimPack. To support the discrete event simulation, continuous simulation, and combined (multi-model) simulation, this is a collection of C/C++ libraries and executable programs. With the help of this program we can construct any type of queueing system and simulation model, and any performance metric can be calculated using any random number generator for the specified random variable. The comparison of the different models and various distributions will be shown through graphical representations.

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