ICAI 2023
12th International Conference on Applied Informatics
Eger, Hungary, 2023

Smooth connection of subdivision curves defined by different subdivision masks

Maja Bódi[a], Ede Troll[b]

[a]Eszterházy Károly Catholic University
bodi.cintia.maja@uni-eszterhazy.hu

[b]Eszterházy Károly Catholic University
troll.ede@uni-eszterhazy.hu

Abstract

Subdivision curves and surfaces are widely used in computer aided geometric design (CAGD). One of the best-known and most commonly used types of curves are defined by Chaikin [1]. With the Lane-Riesenfeld algorithm [4, 5] the B-Spline curves with different degree can be defined. In addition various subdivision curves that have a shape parameter have been defined [2, 3]. Based on the definitions and their parameters the shape of the subdivision curves can be very different.

In this presentation we describe a method of smooth connection of subdivision curves defined by different subdivision masks.

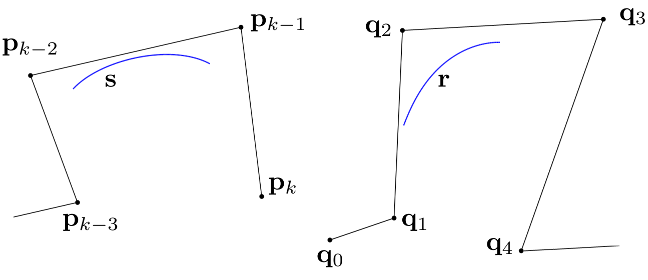

Figure 1. In this picture curve $\mathbf{s}$ is a B-Spline with the degree 3, while the degree of $\mathbf{r}$ is 4 defined by the Lane-Riesenfeld algorithm.

In general, let $\mathbf{s}$ and $\mathbf{r}$ two subdivision curves defined by the Lane-Riesenfeld algorithm and with the control points $\mathbf{p}_i, i \in [0,k]$ and $\mathbf{q}_j, j \in [0,l]$, and their degrees are $n+1$ and $m+1$ respectively where $n \neq m$ and $n,m \geq 2$. In case of n the subdivision (averaging) mask is

$$r_n = \frac{1}{2^n}\left(\binom{n}{0}, \binom{n}{1}, \cdots, \binom{n}{n}\right).$$

Let us consider the function $f_n : [0,n] \to \mathbb{R}$, $f_n(x) = \frac{1}{2^n}\binom{n}{x}$ as the base of the mask r_n and the same way in the case of r_m and f_m. By applying a smooth transition between f_n and f_m we can generate subdivision masks for new arcs which can connect to each other smoothly.

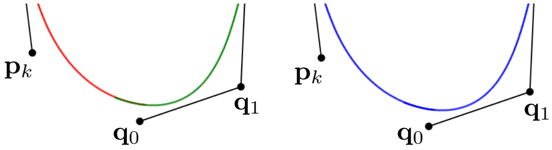

Figure 2. Connection of different curves.

For connected curves, different transitions are used on the functions and masks depending on the degrees of the original curves we would like to join.

References

[1] G. M. Chaikin: *An algorithm for high speed curve generation*, Computer Graphics and Image Processing 3 (1974), pp. 346–349, DOI: https://doi.org/10.1016/0146-664X(74)90028-8.

[2] M. Fang, B. Jeong, J. Yoon: *A family of non-uniform subdivision schemes with variable parameters for curve design*, Applied Mathematics and Computation 313 (2017), pp. 1–11, DOI: https://doi.org/10.1016/j.amc.2017.05.063.

[3] B. Jeong, H. Yang, J. Yoon: *A non-uniform corner-cutting subdivision scheme with an improved accuracy*, Journal of Computational and Applied Mathematics 391 (2021), 113446:1–113446:12, DOI: https://doi.org/10.1016/j.cam.2021.113446.

[4] R. F. Riesenfeld: *On Chaikin's Algorithm*, Computer Graphics and Image Processing 4 (1975), pp. 304–310, DOI: https://doi.org/10.1016/0146-664X(75)90017-9.

[5] R. F. Riesenfeld, J. M. Lane: *A theoretical development for the computer generation and display of piecewise polynomial surfaces*, IEEE Transactions on Pattern Analysis and Machine Intelligence 2.1 (1980), pp. 35–46, DOI: https://doi.org/10.1109/TPAMI.1980.4766968.