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On implicit vector representation of periodic integer-valued signals*

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Abstract

Nowadays, in the field of signal processing, there is a growing interest in non-negative integer-valued time series $\{Y_t\}$ also called discrete or count signal, where Y_t counts the number of occurrences of individuals, objects and events at time t. The count signal analog of the standard autoregressive (AR) model is the integer-valued AR (INAR) model. This model appears as an alternative to the well-known Poisson model family for describing count time series. An INAR model is periodic if its coefficients depends periodically on time.

A non-negative integer ($\mathbb{N}_0$) valued time series $\{Y_t \mid t \in \mathbb{Z}\}$, where $\mathbb{Z}$ denotes the set of integers, is said to be a periodic INAR ($\mathrm{PINAR}_S(\boldsymbol{p})$) process with known period $S \in \mathbb{N}$ and autoregressive orders $\boldsymbol{p} = (p_s) \in \mathbb{N}_0^S$, if it is a solution to the periodic stochastic difference equation

$$Y_{kS+s} = \sum_{i=1}^{p_s} \alpha_{s,i} \circ Y_{kS+s-i} + \varepsilon_{kS+s}, \tag{1}$$

$k \in \mathbb{Z}$, $s = 1, \ldots, S$. In (1), $\alpha_{s,i} \in [0,1]$, $i = 1, \ldots, p_s$, $s = 1, \ldots, S$, are called autoregressive coefficients, the symbol $\circ$ denotes the binomial thinning operator, and the immigration sequence $\{\varepsilon_{kS+s} \mid k \in \mathbb{Z}, s = 1, \ldots, S\}$ consists of $\mathbb{N}_0$-valued random variables. We denote by Y_{kS+s} the series during the sth season of period k. For example, in the case of daily data and weakly periodicity, $S = 7$, s is the day of the week, and k is the index of the week.

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We prove that a PINAR$_S(\boldsymbol{p})$ model can be written, by the help of the matricial binomial thinning operator, in a stationary S-variate INAR form as

$$\boldsymbol{Y}_k = \sum_{i=0}^{p} A_i \circ \boldsymbol{Y}_{k-i} + B \circ \boldsymbol{\varepsilon}_k, \tag{2}$$

where $\boldsymbol{Y}_k := (Y_{kS+1}, Y_{kS+2}, \ldots, Y_{kS+S})^\top$ and $\boldsymbol{\varepsilon}_k := (\varepsilon_{kS+1}, \varepsilon_{kS+2}, \ldots, \varepsilon_{kS+S})^\top$, $k \in \mathbb{Z}$, are forward representations, and $A_0, A_1, \ldots, A_p, B$ are real matrices of $S \times S$ ($\top$ denotes transpose). We suppose that A_0 is a strictly lower triangular matrix. The autoregressive order is defined as $p := [\max_{1\le s\le S}(p_s - s)/S] + 1$, where $[x]$ denotes the greatest integer less than or equal to a real x. The model (2) is called implicit if $A_0 \neq 0$. In this case, $\boldsymbol{Y}_k$ appears on both sides of equation (2). We show that $\{\boldsymbol{Y}_k\}$ can be expressed as a perturbed non-negative VAR$_S(p)$ process in the following form

$$\boldsymbol{Y}_k = \sum_{i=0}^{p} A_i \boldsymbol{Y}_{k-i} + B\boldsymbol{\varepsilon}_k + \boldsymbol{M}_k, \tag{3}$$

where $\{\boldsymbol{M}_k\}$ is a martingale difference sequence with a diagonal conditional heteroscedastic variance matrix which depends on $\boldsymbol{Y}_{k-1}, \ldots, \boldsymbol{Y}_{k-p}$.

We derive necessary and sufficient conditions for the existence of stationary and ergodic solution to models (2) and (3). Moreover, we describe the basic probabilistic properties as the mean, variance and autocovariance function of the above processes.

We estimate the parameters of a PINAR$_S(\boldsymbol{p})$ process by Yule-Walker equations under two scenarios: (i) the immigration random variables are uncorrelated in a period, (ii) the immigration random variables are correlated in a period but the autoregressive orders p_s, $s = 1, \ldots, S$, are identical. The finite sample properties of Yule-Walker estimator are investigated by Monte-Carlo simulation. Finally, we show some applications to the analysis of periodic integer-valued signals.

Keywords. Count signal, binomial thinning, branching process with immigration, periodic correlation, autoregression, martingale difference perturbation, Yule-Walker equations.