

Investigations of alienness of functional equations with computer*

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Abstract

The topic of this talk is connected to computer assisted studies of functional equations. Our aim is to present a package of computer programs, which investigates the so-called alienness and strong alienness properties of two-variable linear functional equations.

The concepts of alienness and strong alienness of functional equations were introduced by J. Dhombres in the following form (cf. [3]). Let $E_1(f) = 0$ and $E_2(f) = 0$ be two functional equations for a function f defined on a nonempty set X and mapping to a groupoid Y with an identity element 0 . The equations E_1 and E_2 are called alien with respect to X and Y , if each solution $f : X \rightarrow Y$ of

$$E_1(f) + E_2(f) = 0$$

is a solution of the system

$$\begin{aligned} E_1(f) &= 0 \\ E_2(f) &= 0. \end{aligned}$$

The equations E_1 and E_2 are said to be strongly alien with respect to X and Y , if all solutions $f, g : X \rightarrow Y$ of

$$E_1(f) + E_2(g) = 0$$

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are solutions of the system

$$\begin{aligned} E_1(f) &= 0 \\ E_2(g) &= 0. \end{aligned}$$

These properties were studied by several authors during the last more than 30 years (cf., e.g., [4] and [8]).

In this talk, we consider them for linear functional equations of the form

$$\sum_{i=0}^{n+1} f_i(p_i x + q_i y) = 0 \quad (x, y \in X), \quad (1)$$

where n is a positive integer, p_0, \dots, p_{n+1} and q_0, \dots, q_{n+1} are rational numbers, X, Y are linear spaces and $f_0, \dots, f_{n+1} : X \rightarrow Y$ are unknown functions. We present a computer program package developed in the computer algebra system Maple (Maple is a trademark of Waterloo Maple Inc.), which is able to decide about the alienness and the strong alienness of functional equations belonging to class (1). The package is based on Maple program, which determines the solutions of linear functional equations of type (1) (cf., [1], furthermore, [2], [5], [6] and [7]).

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