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Bivariate Bernstein Fractal Interpolation Method

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Abstract

The classical methods of real data interpolation can be generalized with fractal interpolation. These fractal interpolation functions provide new methods of approximation of experimental data. These methods can be generalized in multivariate case too. Using a bivariate interpolation formula we will use bivariate Bernstein polynomials and we construct the bivariate fractal interpolation formula and we extend these methods to stochastic case.

A function $f : [a, b] \to \mathbb{R}$, defined on the real interval is named by Barnsley a *fractal function* if the Hausdorff dimension of the graph is noninteger. Barnsley introduced in [2] the notion of a fractal interpolation function (FIF). He said that a fractal function is a (FIF) if it possess some interpolation properties, it means that the fractal function is constrained to go throw on a distinct set of points $(x_i, y_i) \in \mathbb{R}^2, i = 1, 2, ..., N$. In the last few decades the methods of fractal interpolation methods was applied successfully in many fields of applied sciences. It has the advantage that it can be also combine with the classical methods or real data interpolation. Barnsley in [1] introduced the notion of local iterated function systems which are an important generalization of the global iterated function systems. Recently in [3] is introduced a new construction of fractal interpolation of surfaces using bivariate Hermite interpolation. In this article we extend this method in the case of Bernstein polynomials.

1. The two dimensional product-type Bernstein operator on a square

Let D_h be a square with the vertices $V_1 = (0,0), V_2 = (h,0), V_3 = (h,h)$ and $V_4 = (0,h)$ and also F a real-valued function defined on this square. We will consider the following uniform partition on the interval $[0,h]$, regarding with the variable x and y:

$$\Delta_{xm} = \left\{ \frac{i}{m}h, i = \{0, ...m\} \right\}$$

$$\Delta_{yn} = \left\{ \frac{j}{n}h, j = \{0, ...n\} \right\}$$

and the corresponding Bernstein-type operators:

$$(B_{mx}F)(x,y) = \sum_{i=0}^{m} p_{m,i}(x,y)F(\frac{i}{m}h, y)$$

and

$$(B_{ny}F)(x,y) = \sum_{j=0}^{n} q_{n,j}(x,y)F(x, \frac{j}{n}h),$$

where

$$p_{m,i}(x,y) = C_m^i \left(\frac{x}{h}\right)^i \left(1 - \frac{x}{h}\right)^{m-i},$$

respectively

$$q_{n,j}(x,y) = C_n^j \left(\frac{y}{h}\right)^j \left(1 - \frac{y}{h}\right)^{n-j}.$$

Considering the product of these one-dimensional Bernstein operators $P_{m,n} = B_m^x B_n^y$, we have

$$(P_{m,n}F)(x,y) = \sum_{i=0}^{m} \sum_{j=0}^{n} p_{m,i}(x,y) q_{n,j}(x,y) F\left(i\frac{h}{m}, j\frac{h}{n}\right).$$

The product operator $P_{m,n}$ interpolates the domain D_h on the vertices $(P_{m,n}F)(V_i) = F(V_i), \quad i = \{1,..,4\}$, and also we have the following approximation formula

$$F = P_{m,n}F + R_{m,n}F,$$

where $R_{m,n}F$ is the corresponding remainder operator.

2. Generalization of product-type Bernstein approximation formula by fractal interpolation

The product-type Bernstein fractal interpolation function is given by

$$(B^{\alpha,\beta}_{m,n}f)(x,y) = \sum_{i=0}^{m}\sum_{j=0}^{n} B^{\alpha}_{im}(x)B^{\beta}_{jn}(y)f(x_i,y_j), \quad (x_i,y_j) \in D_h$$

where α and β are the corresponding vertical scaling parameters, and $B^{\alpha}_{im}(x)$ and $B^{\beta}_{jm}(x)$ are univariate Bernstein-type fractal interpolation functions, which are constructed with an iterated function system.

Theorem 1. *Let* $0 \le x_0 < x_1 < ... < x_m \le h,\ 0 \le y_0 < y_1 < ... < y_n \le h$, *a set of equidistant data is given, also the value of the function* $f(x_i,y_j), i = 0,1,...,m;\ j = 0,1,...,s$. *The vertical scaling factors* $\alpha_i,\ i = 1,2,...,m$ *and* $\beta_j,\ j = 1,2,...,n$ *such that* $|\alpha|_\infty < 1$, *and* $|\beta|_\infty < 1$. *Than for a fixed* $i, i = 0,1,...,m$ *and* $j, j = 0,1,...,n$ *there exist a fractal function* B^{α}_{im} *and* B^{β}_{jn} *such that*

$$\begin{aligned} B^{\alpha}_{im}(x_i) &= B_{im}(x_i),\ \textit{for all } i = 0,1,...,m \\ B^{\beta}_{js}(y_j) &= B_{js}(y_j),\ \textit{for all } j = 0,1,...,n. \end{aligned}$$

We also study the error of the Bernstein-type bivariate fractal interpolation formula, and we give a delimitation of the error.

References

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