Generate a 3-SAT Problem from a Directed Graph by Algorithm BalatonBoglár

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Abstract

In a previous paper we defined the black-and-white SAT problem which has exactly two solutions, where each variable is either true or false. We showed that black-and-white 2-SAT problems represent strongly connected directed graphs. In this work we would like to extend this result. We introduce Algorihm BalatonBoglár which can generate a 3-SAT problem out of a directed graph. We show that if the input graph is a strongly connected one, than the generated SAT problem is a black-and-white 3-SAT problem. In our previous work the main idea was the following: If a graph contains two edges: $a \to b$, and $a \to c$, then those can be represented by the formula: $(a \supset b) \wedge (a \supset c)$, which is equivalent to two 2-clauses $(\neg a \vee b) \wedge (\neg a \vee c)$. The new idea is the following: If a graph contains two edges: $a \to b$, and $a \to c$, then those can be represented by the formula: $(a \supset b) \vee (a \supset c)$, which is equivalent to $a \supset (b \vee c)$, which is equivalent to a 3-clause $(\neg a \vee b \vee c)$. This idea is not enough to have a black-and-white SAT formula in case of a strongly connected graph. We need to represent simple cycles of the graph, too. If $a_1 \to a_2 \to \cdots \to a_n \to a_1$ is a simple cycle with exit points $b_1, b_2, \ldots b_m$ (which means that for some $i \in \{1 \ldots n\}$, and $j \in \{1 \ldots m\}$ we have $a_i \to b_j$ and $b_j \notin \{a_1, \ldots, a_n\}$), then this cycle can be represented by the clause: $(\neg a_1 \vee \neg a_2 \vee \cdots \vee \neg a_n \vee b_1 \vee b_2 \vee \cdots \vee b_m)$. Algorithm BalatonBoglár uses the trick that instead of detecting each simple cycle it generates from each path $a \to b \to c$ the following 3-clause: $\neg a \vee \neg b \vee c$ even if there is no cycle which contains the vertices a and b. This simplification allows very fast 3-SAT model generation from a directed graph, even the model will be black-and-white 3-SAT if and only if the input directed graph is strongly connected. On the other hand this simplification does not tell us how to generate from a 3-SAT problem a directed graph, unless the 3-SAT problem is black-and-white.

Keywords: SAT, 3-SAT, Black-and-White SAT, Strongly Connected Directed Graph, Algorithm BalatonBoglár

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