

Reflection in quadratic surfaces*

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Abstract

The perspective in Figure 1 shows an interesting phenomenon: The circular boundary c of the plate appears as an ellipse which seems to coincide with the view of the reflection of c in the displayed cup. Is this just by chance or is there a theoretical reason behind?

In the case depicted on the right hand side, the circle c is the focal circle of the reflecting one-sheet hyperboloid. And for this particular case, the displayed phenomenon is a consequence of focal properties of quadratic surfaces. The quadratic tangent cones drawn from a fixed point P to a family of confocal quadrics are confocal and have therefore coinciding axes. For included quadrics passing through P the cones are reducible; two axes lie in their tangent plane while the third axis is the surface normal. Therefore, all focal conics share the property: In the perspective, the images of the curves and their reflections belong to the same conic.

The goal of the lecture is to highlight the geometric background, i.e., to focus on confocal conics and their spatial counterparts with their wide variety of properties (see, e.g., [1, 2]).

Keywords: confocal conics, confocal quadrics, reflection in quadrics

MSC: 51N20, 51N15, 68U05

References

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Figure 1: Why does the bounding circle of the plate continue in the reflection? (photo and computer graphic: Kuno Knöbl, [3])